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Deuteron radial moments for renormalized chiral potentials

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Abstract. We calculate deuteron positive and negative radial moments involving any bilinear function of the deuteron S and D wave functions for renormalized OPE and TPE chiral potentials. The role played by the strong singularities of the potentials at the origin and the short-distance insensitivity of the results when the potentials are fully iterated is emphasized as compared to realistic potentials.

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Chiral dynamics has played an important role in the theoretical description of low-energy hadronic reactions [1] and so far is the only known vestige of the underlying fundamental QCD theory of strong interactions in nuclear physics. There is a number of low-energy theorems based on chiral symmetry which provide a quantitative and model-independent insight into low-energy processes involving pions and nucleons, due to the clear scale separation between nuclear physics and QCD. For compound systems which at low energies disclose their composite nature the theoretical description necessarily becomes very involved and probably dependent on arbitrary assumptions. On the contrary, for weakly bound systems such as the deuterium nucleus one expects important simplifications leading to a more scheme-independent and possibly systematic description of these systems. This possibility motivated the introduction of Effective Field Theory (EFT) approaches [2] for nuclear physics based on the chiral symmetry of QCD, and the derivation of low-energy theorems, as, for example, pion-deuteron scattering [3] (for comprehensive reviews see, *e.g.*, refs. [4–6]). In many cases most of the information needed for reactions involving the deuteron can be encoded by simple deuteron matrix elements.

Guided by earlier work [7,8], we have proposed [9–13] to renormalize the NN interaction in a non-perturbative way, highlighting model-independent long-distance correlations among physical observables. In our approach the long-distance chiral NN One-Pion Exchange (OPE) and Two-Pion Exchange (TPE) potentials, computed within perturbation theory in refs. [14–16], are iterated to all orders in the Schrödinger equation very much in the spirit of the original Weinberg approach [2]. However, some subtleties are found [10,12,17], which impose strong constraints on the admisible short-distance physics based on orthogonality, uniqueness and finiteness of the results.

In the ${}^3S_1 - {}^3D_1$ channel, the relative proton-neutron state for negative energy is described by the coupled equations

$$\begin{pmatrix} -\frac{\mathrm{d}^2}{\mathrm{d}r^2} + U_s(r) & U_{sd}(r) \\ U_{sd}(r) & -\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{6}{r^2} + U_d(r) \end{pmatrix} \begin{pmatrix} u \\ w \end{pmatrix} = - \gamma^2 \begin{pmatrix} u \\ w \end{pmatrix}. \qquad (1)$$

Here $\gamma = \sqrt{MB}$, with B the deuteron binding energy and M the nucleon mass, $U(r) = MV(r)$ are the reduced potentials and $u(r)$ and $w(r)$ are S- and D-wave deuteron reduced wave functions, respectively. At long distances they satisfy,

$$\begin{pmatrix} u \\ w \end{pmatrix} \to A_S \, e^{-\gamma r} \begin{pmatrix} 1 \\ \eta \left[1 + \frac{3}{\gamma r} + \frac{3}{(\gamma r)^2} \right] \end{pmatrix}, \qquad (2)$$

where η is the asymptotic D/S ratio parameter and A_S is the asymptotic normalization factor, which is such that the deuteron wave functions are normalized to unity. For conventions and numerical values of parameters we use refs. [11,12] throughout.

In this work we report on the radial moments

$$\langle r^n \rangle_u = \int_0^\infty r^n u(r)^2 \mathrm{d}r, \qquad (3)$$

$$\langle r^n \rangle_w = \int_0^\infty r^n w(r)^2 \mathrm{d}r, \qquad (4)$$

$$\langle r^n \rangle_{uw} = \int_0^\infty r^n u(r) w(r) \mathrm{d}r, \qquad (5)$$

for $-3 \le n \le 2$ which appear in many situations of interest, such as the calculation of the matter radius, the

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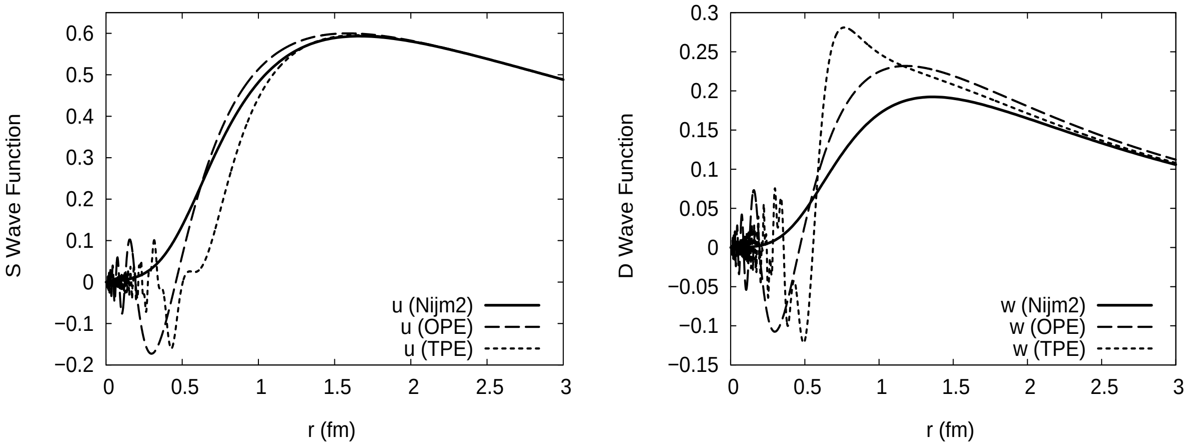

Fig. 1. The OPE and TPE deuteron wave functions, u (left) and w (right), as a function of the distance (in fm) compared to the Nijmegen-II wave functions [22]. The asymptotic normalization $u \to e^{-\gamma r}$ has been adopted and the asymptotic D/S ratio is taken $\eta = 0.0256(4)$ in the TPE case (for OPE $\eta = 0.026333$). We use the set IV of chiral couplings (see ref. [12]).

deuteron quadrupole moment and deuteron magnetic moment for the positive powers, as well as πd and Kd elastic scattering and neutral pion photoproduction, $\gamma d \to \pi^0 d$, in the case of the negative powers.

An important issue is the finiteness of the negative radial moments, a topic which has been recently discussed for OPE [11,18,19] and TPE [20]. The remarkable finding is that chiral potentials [14–16], when fully iterated, have an increasing number of finite inverse radial moments due to the near the origin singularities of the potential. They smoothen the short-distance behaviour of the wave functions and hence improve the convergence of the inverse radial moments. This is in sharp contrast with perturbative approaches [21], for which the perturbative wave functions diverge at the origin [11,12][1], or conventional (regular) phenomenological potentials [22] where the S- and D-wave short-distance behaviour of the wave functions, $u \sim r$ and $w \sim r^5$ respectively, is enough to render $\langle 1/r \rangle_u$ and $\langle 1/r^2 \rangle_u$ finite, but produce divergent higher inverse moments. We illustrate the situation below.

At distances much shorter than the pion Compton wavelength, the OPE potential behaves as

$$\begin{pmatrix} U_s^{\rm OPE}(r) & U_{sd}^{\rm OPE}(r) \\ U_{sd}^{\rm OPE}(r) & U_d^{\rm OPE}(r) \end{pmatrix} \to \frac{1}{r^3} \begin{pmatrix} R_s & R_{sd} \\ R_{sd} & R_d \end{pmatrix} \qquad (6)$$

with $R_s = 0$, $R_{sd} = 2\sqrt{2}R$, $R_d = 4R$ and $R = 3g_A^2 M/32\pi f^2$ $(= 1.07764\,\text{fm})$. This behaviour of the potential is strong enough to overcome the centrifugal barrier at short distances, thus modifying the usual short-distance behaviour of the wave functions, which can schematically be written as

$$u(r) \sim w(r) \sim \left(\frac{r}{R}\right)^{3/4} f\left(\frac{r}{R}\right), \qquad (7)$$

where $f(r/R)$ represents some linear combination of $\sin(4\sqrt{R/r})$, $\cos(4\sqrt{R/r})$ and $\exp(-4\sqrt{2}\sqrt{R/r})$ (for a complete analysis, see ref. [11]). The elimination of the diverging exponential fixes $\eta_{\rm OPE} = 0.0263$. From this short-distance behaviour of the wave functions, one finds that the $\langle 1/r \rangle_u$ and $\langle 1/r^2 \rangle_u$ moments are finite for the OPE potential, while $\langle 1/r^3 \rangle_u$ and higher moments diverge, as it would happen for a regular potential.

The short-distance behaviour of the TPE (NNLO) has been exploited in ref. [12]. The potential at short distances behaves as [14–16]

$$\begin{pmatrix} U_s^{\rm TPE}(r) & U_{sd}^{\rm TPE}(r) \\ U_{sd}^{\rm TPE}(r) & U_d^{\rm TPE}(r) \end{pmatrix} \to \frac{1}{r^6} \begin{pmatrix} R_s^4 & R_{sd}^4 \\ R_{sd}^4 & R_d^4 \end{pmatrix}, \qquad (8)$$

where

$$\begin{aligned} (R_s)^4 &= \frac{3g_A^2}{128 f^4 \pi^2}(4 - 3g_A^2 + 24\bar{c}_3 - 8\bar{c}_4), \\ (R_{sd})^4 &= -\frac{3\sqrt{2}g_A^2}{128 f^4 \pi^2}(-4 + 3g_A^2 - 16\bar{c}_4), \\ (R_d)^4 &= \frac{9g_A^2}{32 f^4 \pi^2}(-1 + 2g_A^2 + 2\bar{c}_3 - 2\bar{c}_4), \end{aligned} \qquad (9)$$

and $\bar{c}_i = M c_i$ are the low-energy chiral couplings appearing in πN scattering. As in the OPE case, this potential is strong enough at short distances to modify the short-distance behaviour of the wave function, which now reads

$$\begin{aligned} u(r) \sim w(r) \sim\; & C_+ \, (r/R_+)^{3/2} f_+ (r/R_+) \\ & + C_- \, (r/R_-)^{3/2} f_- (r/R_-), \end{aligned} \qquad (10)$$

where R_+^4 and R_-^4 are the eigenvalues of the matrix in eq. (8), and $f_\pm(r/R_\pm)$ represents a linear combination of $\sin(R_\pm^2/2r^2)$ and $\cos(R_\pm^2/2r^2)$ leaving $\eta_{\rm TPE}$ as a free parameter [12,20]. From this short-distance behaviour, the $\langle 1/r \rangle_u$, $\langle 1/r^2 \rangle_u$ and $\langle 1/r^3 \rangle_u$ radial moments are finite for the TPE potential, while higher moments diverge (although they would become finite for higher-order potentials).

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[1] This divergency holds for perturbations both on boundary conditions or on distorded (fully iterated) OPE waves.

Table 1. Deuteron radial moments (in units of powers of fm). We consider the OPE and TPE potentials; in the case of the OPE potential we have taken $g_{\pi NN} = 13.08$ (*i.e.*, $g_A = 1.29$, OPE) and $g_A = 1.26$ (OPE*), while in the TPE case we show the results corresponding to the four set of chiral couplings considered along our previous works [12,20]. In the OPE case the error is estimated by varying the semiclassical matching radius [11,20] in the 0.1–0.2 fm range, while in the TPE case the error comes from the experimental uncertainty of the D/S ratio, $\eta = 0.0256(4)$. TPE Sets I, II, II and IV refer to the chiral parameters, c_1, c_3 and c_4 of refs. [23,16,24] and [25], respectively. NijmII and Reid93 are calculated from ref. [22] or taken from ref. [26].

	Short	OPE	OPE*	TPE-SetI	TPE-SetII	TPE-SetIII	TPE-SetIV	NijmII	Reid93
γ (fm^{-1})	Input	Input	Input	Input	Input	Input	Input	0.231605	0.231605
η	0.0	0.026333	0.025547	Input	Input	Input	Input	0.02521	0.02514
$\langle r^2\rangle_u$	9.3213	14.582(6)	14.424(6)	15.60(9)	15.61(11)	15.3(3)	15.09(13)	15.129	15.147
$\langle r^2\rangle_w$	0.0	0.3849(2)	0.36371(15)	0.37(3)	0.38(2)	0.37(2)	0.38(2)	0.3438	0.3429
$\langle r^2\rangle_{uw}$	0.0	2.0883(9)	2.0144(8)	2.14(4)	2.15(3)	2.11(3)	2.09(3)	2.035	2.032
$\langle r^1\rangle_u$	2.1589	3.0400(14)	3.0185(13)	3.21(2)	3.20(2)	3.16(2)	3.12(3)	3.138	3.139
$\langle r^1\rangle_w$	0.0	0.14287(6)	0.13691(6)	0.134(12)	0.139(12)	0.136(12)	0.146(12)	0.1204	0.1206
$\langle r^1\rangle_{uw}$	0.0	0.5898(3)	0.5739(2)	0.586(14)	0.589(14)	0.581(15)	0.584(13)	0.5594	0.5590
$\langle r^0\rangle_u$	1.0	0.9270(4)	0.9287(4)	0.935(9)	0.930(9)	0.931(10)	0.918(10)	0.9436	0.9430
$\langle r^0\rangle_w$	0.0	0.07312(3)	0.07146(2)	0.065(9)	0.070(9)	0.069(10)	0.081(10)	0.05635	0.05699
$\langle r^0\rangle_{uw}$	0.0	0.23989(11)	0.23691(10)	0.222(7)	0.225(7)	0.225(8)	0.233(7)	0.2166	0.2172
$\langle r^{-1}\rangle_u$	∞	0.4259(3)	0.4336(2)	0.382(5)	0.377(5)	0.388(6)	0.384(5)	0.4160	0.4163
$\langle r^{-1}\rangle_w$	0.0	0.052498(5)	0.05256(3)	0.042(8)	0.048(7)	0.048(10)	0.063(10)	0.03419	0.03520
$\langle r^{-1}\rangle_{uw}$	0.0	0.14120(7)	0.14239(3)	0.112(5)	0.115(5)	0.117(6)	0.128(6)	0.1153	0.1166
$\langle r^{-2}\rangle_u$	∞	0.3464(8)	0.3582(3)	0.210(4)	0.205(4)	0.220(5)	0.221(3)	0.2607	0.2646
$\langle r^{-2}\rangle_w$	0.0	0.0771(2)	0.0783(3)	0.038(8)	0.044(7)	0.045(12)	0.064(11)	0.02613	0.02780
$\langle r^{-2}\rangle_{uw}$	0.0	0.1551(4)	0.1589(3)	0.072(4)	0.075(4)	0.079(4)	0.093(6)	0.08122	0.08413
$\langle r^{-3}\rangle_u$	∞	∞	∞	0.159(3)	0.155(3)	0.173(4)	0.1851(8)	∞	∞
$\langle r^{-3}\rangle_w$	0.0	∞	∞	0.053(10)	0.059(9)	0.066(14)	0.091(14)	0.02465	0.02783
$\langle r^{-3}\rangle_{uw}$	0.0	∞	∞	0.0626(14)	0.068(2)	0.071(2)	0.097(6)	0.07342	0.08064

The wave functions for OPE and TPE as compared to the NijmII ones have been depicted in fig. 1. The radial moments are tabulated in table 1. As we see, and despite the very different behaviour at short distances between the deuteron wave functions corresponding to renormalized chiral potentials and to phenomenological potentials, the convergent moments are fairly similar (the $u+w$ combination works better) despite that NijmII and Reid93 contain no explicit TPE components. For inverse moments the trend improves clearly when going from OPE to TPE, which we interpret as a correct implementation of model-independent long-distance correlations generated by chiral symmetry and renormalization constraints.

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